

Networks, Phillips Curves, and Monetary Policy

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Macro Reading Group

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New Keynesian Model

- What is the source of inflation?
- What are the sources of distortion?
 - Market Power
 - Staggered Pricing
 - 1 Average markup is different from the desired value.
 - 2 Relative price differences.

Review - NK Philips Curve and 'Divine Coincidence'

Phillips Curve: $\pi_t = \beta E_t \pi_{t+1} + \alpha(y - y^*)$

Divine coincidence: In this model stabilizing output gap stabilizes inflation and vice versa. Achieving one of the objective achieves the other as well. No trade-off!

Reason: Shocks affect both y and y^* equally.

Implication: A single mandate on price stability or output stability has the same result as a dual mandate on maintaining them together.

Does it really hold?: Well, simply put NO! It is theoretically true only because of a set of restrictive assumptions. Breaks down in presence of firm level dynamics, wage rigidities etc etc.

Okay! So, What's Rubbo proposing in the paper?

- She derives a generalised version of Philips curve in a disaggregated economy, with multiple sectors and a general input-output network.
 - This endogenously does away with the divine coincidence.
 - And results in a flatter Phillips curve.
- She goes further and creates an inflation index (combining sector level indices) which restores the divine coincidence!
- Finally, she derives the welfare function and the optimal policy in this model.

And how?

- **Step 1:** Derive, at the sectoral level, the response of inflation to the output gap and productivity.
- **Step 2:** Aggregate sector level responses to a Phillips curve.
 - In this way, a Philips curve can be characterised associated with an given inflation index. Different indices are constructed using different sector level weights.

Before we begin - All the parameters at one place

- $N \rightarrow$ Total number of **sectors**.
- $\beta \rightarrow N \times 1$ vector denoting **consumption shares**. $\beta_i = \frac{p_i c_i}{PC}$
- $\alpha \rightarrow N \times 1$ vector denoting **labour shares** in total sales. $\alpha_i = \frac{wL_i}{p_i y_i}$
- $\Omega \rightarrow N \times N$ **Input output matrix**. $\omega_{ij} = \frac{p_j x_{ij}}{p_i y_i}$
- $\lambda \rightarrow N \times 1$ vector of **sales shares**. $\lambda_i = \frac{P_i Y_i}{PC}$
- $\epsilon_i \rightarrow$ elasticity of substitution **between varieties from sector i**.
- $\theta_{ij}^k \rightarrow$ elasticity of substitution **between goods i and j in the production of k**.
- σ_{ij}^C elasticity of substitution **between goods i and j in consumption**.
- θ_{iL}^k elasticity of substitution **between good i and Labor in the production of good k**

Utility

$$U = \frac{C^{1-\gamma}}{1-\gamma} - \frac{L^{1+\phi}}{1+\phi} \quad (1)$$

where C is the constant returns to scale aggregator of all the goods produced in the economy and L is the total labor supply.

Constraints

Cash advance and budget constraint

$$PC \leq M \quad (2)$$

$$PC \leq wL + \Pi - T \quad (3)$$

Production Function

N sectors. Within each sector a continuum of firms, producing differentiated goods, with a CRS production function.

$$Y_i = A_i F_i(L_i, x_{ij}) \quad (4)$$

x_{ij} is the quantity of good j used as the input and A_i is the Hicks neutral sector specific productivity shock.

Cost minimization: $\min_{\{x_{ij}\}, L_i} wL_i + \sum_j p_j x_{ij}$ s.t. $A_i F_i(L_i, x_{ij}) = \bar{y}$

Price setting: $\max_{p_i} D_i(p_i - (1 - \tau_i)mc_i) \left(\frac{p_i}{P_i}\right)^{-\epsilon_i}$

Input subsidies are set so as to exactly offset the markup distortion i.e.

$$1 - \tau_i = \frac{\epsilon_i - 1}{\epsilon_i}$$

Philips Curve

- $\mathcal{B} \rightarrow N \times 1$ vector, \mathcal{B}_i is the elasticity of sector i 's price w.r.t the output gap.
- $\mathcal{V} \rightarrow N \times N$ matrix, \mathcal{V}_{ij} is the elasticity of sector i 's price with respect to a productivity shock to sector j .

$$\underbrace{\pi}_{N \times 1} = \underbrace{\mathcal{B}}_{N \times 1} \tilde{y} + \underbrace{\mathcal{V}}_{N \times N} \underbrace{d\log A}_{N \times 1} \quad (5)$$

Combine sector level inflation rates to get aggregate inflation.

$$\pi^{AGG} \equiv \phi^T \pi = \sum_i \phi_i \pi_i \quad (6)$$

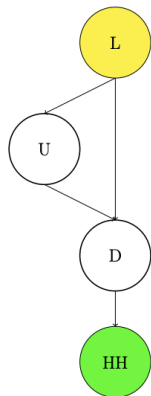
Or simply

$$\pi^{AGG} = \phi^T \underbrace{\mathcal{B}}_{N \times 1} \tilde{y} + \phi^T \underbrace{\mathcal{V}}_{N \times N} \underbrace{d\log A}_{N \times 1} \quad (7)$$

Consumer inflation, a special case, where $\phi = \beta$

Example 1 - Vertical Chain Economy

- Two Sectors: U (Upstream) and D (Downstream)
- Both use Labour
- Only D sells the final good.



Leontief Inverse

Let sector i produce x_i units of good.

And sector j uses ω_{ij} proportion of its output as an input from sector i for production. And let d_i be the consumer demand. Then

$$x_i = \omega_{i1}x_1 + \omega_{i2}x_2 + \dots + d_i \quad (8)$$

In matrix notation

$$x = \Omega x + d \quad (9)$$

$$x = (I - \Omega)^{-1}d \quad (10)$$

In the paper, labor is the only factor of production so

$$\alpha_i + \sum_j \omega_{ij} = 1 \quad (11)$$

$$\alpha + \Omega \mathbf{1} = \mathbf{1} \implies (I - \Omega)^{-1} \alpha = \mathbf{1}$$

Adjusted Leontief Inverse

Leontief inverse captures the direct and indirect expenditure of sector i on goods from sector j , as a share of i 's revenue.

With flexible prices this is also the elasticity of i 's marginal cost to j 's marginal cost, because firms charge constant markups. With price rigidities this is no longer true, as marginal cost changes do not get fully passed-through into input prices. In this case the "direct" elasticity of i 's marginal cost with respect to j 's is $\omega_{ij}\delta_j$ which discounts ω_{ij} by the number of firms who can change their price.

The adjusted Leontief Inverse is thus given by

$$(I - \Omega\Delta)^{-1} \tag{12}$$

Hulten's Theorem

In an efficient economy, the macro impact of a shock to industry i depends on i 's sales as a share of aggregate output, up to a first-order approximation.

$$d\log A_{AGG} = \lambda^T d\log A \quad (13)$$

From the consumption leisure tradeoff

$$d\log L^{nat} = \frac{1 - \gamma}{\gamma + \phi} \lambda^T d\log A \quad (14)$$

$$y^{nat} = d\log L^{nat} + d\log A_{AGG} = \frac{1 + \phi}{\gamma + \phi} \lambda^T d\log A \quad (15)$$

Lastly,

$$\tilde{y} = d\log L - d\log L^{nat} = d\log L - \frac{1 - \gamma}{\gamma + \phi} \lambda^T d\log A \quad (16)$$

Example 1 - Vertical Chain Economy

$$\mathcal{V} = \begin{bmatrix} \mathcal{V}_{UU} & \mathcal{V}_{UD} \\ \mathcal{V}_{DU} & \mathcal{V}_{DD} \end{bmatrix} \quad \Delta = \begin{bmatrix} \delta_U & 0 \\ 0 & \delta_D \end{bmatrix} \quad \Omega = \begin{bmatrix} \omega_{UU} & \omega_{UD} \\ \omega_{DU} & \omega_{DD} \end{bmatrix} \equiv \begin{bmatrix} 0 & 0 \\ 1 & 0 \end{bmatrix} \quad \beta = \begin{bmatrix} 0 \\ 1 \end{bmatrix}$$

$$\alpha = \begin{bmatrix} 1 - \alpha_D \\ \alpha_D \end{bmatrix}$$

Adjusted Leontief Inverse: $(I - \Omega\Delta)^{-1} \equiv \begin{bmatrix} 1 & \delta_D \\ 0 & 1 \end{bmatrix}$

Example 1 - Vertical Chain Economy

Consider a negative productivity shock that hits D.

Using the calvo pricing assumption: $dlogp = \Delta dlogmc$ and the **consumer price change** is simply $dlogP = \beta^T \Delta dlogmc$ i.e.

$$dlogP = \begin{bmatrix} 1 & 0 \end{bmatrix} \begin{bmatrix} \delta_U & 0 \\ 0 & \delta_D \end{bmatrix} \begin{bmatrix} dlogmc_U \\ dlogmc_D \end{bmatrix} \quad (17)$$

$$\implies dlogP = \delta_D dlogmc_D$$

This makes sense as consumers buy only from sector D. So, the inflation is only driven by a change in marginal costs of sector D.

Okay, so what is the change in Marginal cost then?

$$dlogmc_D = \alpha_D dlogw + \sum_{j \in \{U, D\}} \omega_{Dj} dlogp_j - dlogA_D \quad (18)$$

$$dlogmc_D = \alpha_D dlogw + dlogp_U - dlogA_D \quad (19)$$

Example 1 - Vertical Chain Economy

Using the Calvo assumption again and re-arranging, we get

$$d\log mc_D = \alpha_D d\log w + \delta_U d\log mc_U - d\log A_D \quad (20)$$

Similarly the change in marginal cost in Sector D is given by (it's all because of the change in wages)

$$d\log mc_U = (1 - \alpha_D) d\log w \quad (21)$$

Lastly, from the consumption leisure trade-off

$$\begin{aligned} d\log w &= d\log P + \phi d\log L + \gamma d\log y \\ d\log w &= d\log P + (\gamma + \phi)\tilde{y} + d\log A_D \end{aligned} \quad (22)$$

and obviously $d\log P = d\log mc_D$.

Example 1 - Vertical Chain Economy

Solving the system of equation (??), (??) and (??), we get

$$d\log w = \frac{(\gamma + \phi)\tilde{y} + (1 - \delta_D)d\log A_D}{1 - \bar{\delta}_w} \quad (23)$$

$$d\log mc_U = (1 - \alpha_D)\frac{(\gamma + \phi)\tilde{y}}{1 - \bar{\delta}_w} + \frac{(1 - \delta_D)d\log A_D}{1 - \bar{\delta}_w} \quad (24)$$

$$d\log P = \delta_D d\log mc_D = \frac{\bar{\delta}_w(\gamma + \phi)}{1 - \bar{\delta}_w}\tilde{y} + \left(\bar{\delta}_w\frac{1 - \delta_D}{1 - \bar{\delta}_w} - \delta_D\right)d\log A_D \quad (25)$$

where

$$\bar{\delta}_w = \delta_D(\alpha_D - (1 - \alpha_D)\delta_U) \quad (26)$$

Vertical Chain Economy

(??) is the Philips curve and the elasticities \mathcal{V}_{UD} and \mathcal{V}_{DD} are

$$\mathcal{V}_{UD} = \delta_U \frac{1 - \delta_D}{1 - \bar{\delta}_w} \quad (27)$$

$$\mathcal{V}_{DD} = \bar{\delta}_w \frac{1 - \delta_D}{1 - \bar{\delta}_w} - \delta_D \quad (28)$$

$$\mathcal{B}_U = \frac{1 - \alpha_D}{1 - \bar{\delta}_w} (\gamma + \phi) \quad (29)$$

$$\mathcal{B}_D = \frac{\bar{\delta}_w}{1 - \bar{\delta}_w} (\gamma + \phi) \quad (30)$$

Slope of the Phillips curve

\mathcal{B}_D has two components

- $(\gamma + \phi) \rightarrow$ depends on the parameters of the labor supply curve and captures the elasticity of real wages w.r.t the output gap.
- $\frac{\bar{\delta}_w}{1 - \bar{\delta}_w} \rightarrow$ gives the pass through of real wages into prices. The numerator is the pass through of the nominal wages and the denominator is the general equilibrium effect.

Had there been only one sector, D, then $\alpha = 1$ and $\bar{\delta}_w = \delta_D$. This implies that the wage pass through only depends upon the price rigidity parameter.

BUT $\bar{\delta}_w \leq \delta_D$ i.e. the network structure dampens the pass through effect and results in a flatter Phillips curve.

WHY? Because of the intermediate goods, now the wage pass through in D depends on its labour share as well as on U labour share (which is also imperfect due to price rigidities.)

Again two components

- The change in real wages due the productivity shock is given $d\log A_D$. The general equilibrium effect $\frac{1-\delta_D}{1-\bar{\delta}_w}$ maps it to the nominal wages. And because of the network structure it is dampened by $\bar{\delta}_w$. So the wage component $= \bar{\delta}_w \frac{1-\delta_D}{1-\bar{\delta}_w}$
- The productivity also affects the marginal cost directly, and indirectly through input prices i.e. $= -d\log A_D$. To map marginal costs into prices, just multiply by adjustment probability δ_D . Summing both these up, we get:

$$\mathcal{V}_{DD} = \bar{\delta}_w \frac{1 - \delta_D}{1 - \bar{\delta}_w} - \delta_D \quad (31)$$

NOTICE - Up till the time these two components - wage and productivity effects are different - inflation is never stabilised; even under no output gap.

Why does the divine coincidence not hold?

In a Hicks neutral shock, what is the pass through of the productivities?

$$\bar{\delta}_a = \delta_D(1 + (1 - \alpha_D)\delta_U) \quad (32)$$

This implies that $\bar{\delta}_a \geq \delta_w$ and hence consumer inflation is positive.

But we saw that the prices in sector U actually fell down. Are they being captured? No! Because we use consumption shares to weigh sector inflation, and it assigns no weight to the prices of the upstream sector. Hence, consumer inflation is not stabilised.

And the Divine coincidence doesn't hold anymore.

Consumer inflation is not stabilized because it fails to account for the decrease in upstream prices.

Generalised Results

$$\mathcal{B} = \frac{\Delta(I - \Omega\Delta)^{-1}\alpha}{1 - \bar{\delta}_w}(\gamma + \phi)$$

where

(33)

$$\bar{\delta}_w \equiv \beta^T \Delta(I - \Omega\Delta)^{-1}\alpha$$

is the pass-through of nominal wages into consumer prices.

$$\mathcal{V} = \Delta(I - \Omega\Delta)^{-1} \left[\frac{1 - \bar{\delta}_a}{1 - \bar{\delta}_w} \alpha \lambda^T - I \right]$$

where

(34)

$$\bar{\delta}_a(d\log A) \equiv \frac{\beta^T \Delta(I - \Omega\Delta)^{-1} d\log A}{\lambda^T d\log A}$$

is the pass-through of the productivity shock into consumer prices, relative to the aggregate shock.

Slope

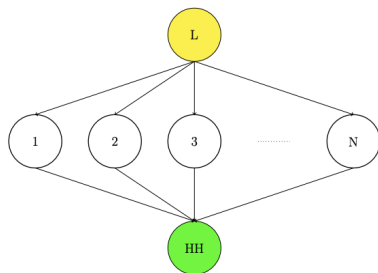
$$\kappa^C = \frac{\bar{\delta}_w}{1 - \bar{\delta}_w} (\gamma + \phi) \quad (35)$$

Residual

$$u^C = \frac{\bar{\delta}_w - \bar{\delta}_a}{1 - \bar{\delta}_w} \lambda^T d \log A \quad (36)$$

Intuition: The residual depends upon the relative pass through of the wages and productivity. In the baseline model they cancel each other out and it results in no residuals.

Example 2 - Horizontal Economy



Example 2 - Horizontal Economy

The inflation in each sector is given by

$$\pi_i = \delta_i \left(\frac{1 - \bar{\delta}_A}{1 - \bar{\delta}_w} - d\log A_i \right) \quad (37)$$

where

$$\bar{\delta}_w = \mathbb{E}_\beta(\delta) \quad (38)$$

$$\bar{\delta}_A = \frac{\mathbb{E}_\beta(\delta d\log A)}{\mathbb{E}_\beta(d\log A)} \quad (39)$$

Lastly, consumer inflation is given by

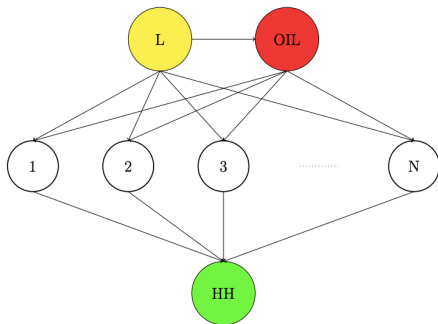
$$\pi^C = -\frac{\text{Cov}_\beta(\delta, d\log A)}{1 - \mathbb{E}_\beta(\delta)} \quad (40)$$

Example 2 - Horizontal Economy - Divine Coincidence

If productivity increases more than the average in more flexible sectors (and vice versa) then the covariance is positive and hence the consumer inflation is negative.

A productivity shock of same size in a sector with less flexible prices weighs less in the consumer inflation. Hence the consumer price inflation overrepresents sectors with more flexible prices.

Example 3 - Oil Economy



Example 3 - Oil Economy

$$\pi^C = -\frac{\text{Cov}_\beta(\delta, \omega_{oil}) + (1 - \delta_L)\mathbb{E}_\beta(\delta)\mathbb{E}_\beta(\omega_{oil})}{1 - \delta_L\mathbb{E}_\beta(\delta)}d\log A_{oil} \quad (41)$$

- Input-Output linkages reduce the slope of the consumer-price Phillips curve.
- Productivity fluctuations result in endogenous cost-push shocks.
- Divine Coincidence doesn't hold. Because
 - consumption shares do not capture the contribution of upstream sectors to value added
 - consumer prices fail to account for the fact that flexible sectors respond more to a given cost shock

Next?

- She derives a generalised version of Philips curve in a disaggregated economy, with multiple sectors and a general input-output network.
 - This endogenously does away with the divine coincidence.
 - And results in a flatter Phillips curve.
- She goes further and creates an inflation index (combining sector level indices) which restores the divine coincidence!
- Finally, she derives the welfare function and the optimal policy in this model.

The following relation holds between the output gap and sector-level markups:

$$(\gamma + \phi)\tilde{y} = \lambda^T d\log\mu \quad (42)$$

and from the pricing equation

$$-d\log\mu = (I - \Delta)\Delta^{-1}\pi \quad (43)$$

So, the sales weighted Phillips curve is simply given by

$$SW = \lambda^T(I - \Delta)\Delta^{-1}\pi = -\lambda^T d\log\mu = (\gamma + \phi)\tilde{y} \quad (44)$$

SW is the only aggregate inflation statistic such that the corresponding Phillips curve has no endogenous residual.

The weights in SW are all positive, therefore the aggregate can only be zero if π_i is positive in some sectors and negative in other sectors

Phillips curve regressions

Time period: January 1984 - July 2018 Calibrated slopes: SW = -3 and Consumer Prices = -0.09

	SW	CPI	core CPI	PCE	core PCE
gap	-3.8814** (0.6329)	-0.2832** (0.0729)	-0.1839** (0.0642)	-0.1667** (0.0628)	-0.1007* (0.0565)
intercept	1.9842** (0.0475)	2.9052** (0.1196)	2.9021** (0.1052)	2.3978** (0.103)	2.372** (0.0926)
R-squared	0.2154	0.0991	0.0566	0.0489	0.0227

Table 5: CBO unemployment gap

	SW	CPI	core CPI	PCE	core PCE
gap	-1.1054** (0.3275)	-0.1613** (0.0809)	-0.0344 (0.052)	-0.062 (0.0487)	0.0047 (0.0368)
inflation expectations	0.8287** (0.0383)	0.4846** (0.1557)	0.5446** (0.0559)	0.6364** (0.0621)	0.6406** (0.045)
intercept	0.3484** (0.0789)	1.3851** (0.5021)	1.3193** (0.1818)	0.5522** (0.196)	0.8388** (0.1228)
R-squared	0.8234	0.159	0.4425	0.4635	0.6072

Table 6: CBO unemployment gap

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Welfare loss in the baseline one sector model

$$\mathbb{W} = \frac{1}{2} \left[(\gamma + \phi) \tilde{y}^2 + \epsilon \frac{1 - \delta}{\delta} \pi^2 \right] \quad (45)$$

But now, the second-order welfare loss with respect to the flex-price efficient outcome is:

$$\mathbb{W} = \frac{1}{2} [(\gamma + \phi) \tilde{y}^2 + \pi^T \mathcal{D} \pi] \quad (46)$$

Where the matrix \mathcal{D} can be decomposed into $\mathcal{D}_1 + \mathcal{D}_2$, where \mathcal{D}_1 captures the productivity loss from within sector misallocation and \mathcal{D}_2 captures the productivity loss from cross sector misallocation.

Welfare loss due to two inefficiencies

- **Within sector misallocation:** Same as the baseline model, relative price distortion between adjusting and non adjusting firms. Customers buy too much of the variety whose price is relatively lower as compared to the efficient equilibrium.

$$\pi^T \mathcal{D}_1 \pi = \sum_i \lambda_i \frac{1 - \delta_i}{\delta_i} \quad (47)$$

- **Cross sector misallocation:** Relative price distortions induce producers in each sector t to substitute towards the inputs whose relative price is lower than in the efficient equilibrium.

$$\pi^T \mathcal{D}_2 \pi = \sum_t \lambda_t \sum_{i,j} \Phi_t(i,j) \quad (48)$$

Cross Sector Misallocation

Steps

- Isolate the distortionary component of sectoral inflation rates and track its propagation across the network, which results in relative price distortions across t 's inputs.
- Translate relative price distortions into t 's productivity loss from inefficient substitution. This is given by the substitution operators.

NOTE: Inflation is associated with a distortion only to the extent that it mirrors the change in the markup of non-adjusting firms.

And mapping between μ and π is given by: $-d\log\mu = (I - \Delta)\Delta^{-1}\pi$

Finally, distortion in the relative price of a sector k can come either directly from a change in k 's markup, or indirectly from a change in the markup of some of its inputs. Thus

$$d\log p - d\log w = (I - \Omega)^{-1}(I - \Delta)\Delta^{-1}\pi \quad (49)$$

Cross Sector Misallocation

Propogation across the network: The relative price distortion between each sector pair (k, h) associated with inflation in sector i.

$$((I - \Omega)_{ki}^{-1} - (I - \Omega)_{hi}^{-1}) \frac{1 - \delta_i}{\delta_i} \pi_i \quad (50)$$

The productivity effect in sector t is captured by the corresponding substitution, and depends on the interaction between distortions associated with inflation in different sectors. Intuitively, distortions from π_i and π_j reinforce each other if they produce similar relative price changes across input pairs (k, h), especially those with higher input shares or higher elasticity of substitution.

$$\Phi_t(i, j) = \omega_{tk} \omega_{th} \theta_{kh}^t ((I - \Omega)_{ki}^{-1} - (I - \Omega)_{hi}^{-1}) \frac{1 - \delta_i}{\delta_i} \pi_i ((I - \Omega)_{kj}^{-1} - (I - \Omega)_{hj}^{-1}) \frac{1 - \delta_j}{\delta_j} \pi_j \quad (51)$$

$$\begin{aligned} \min_{\tilde{y}, \pi} \frac{1}{2} [(\gamma + \phi)\tilde{y}^2 + \pi^T \mathcal{D}\pi] \\ \text{s.t. } \pi = \mathcal{B}\tilde{y} + \mathcal{V}d\log A \end{aligned} \quad (52)$$

and solving we get the value of output gap that minimizes the loss as

$$\tilde{y}^* = -\frac{\mathcal{B}^T \mathcal{D} \mathcal{V} d\log A}{\gamma + \phi + \mathcal{B}^T \mathcal{D} \mathcal{B}} \quad (53)$$

Inflation targeting

Then there exists a unique vector of weights ϕ (up to a multiplicative constant) such that the aggregate inflation $\pi_\phi = \phi^T \pi$, which is given by $\phi^T = \lambda^T (I - \Delta) \Delta^{-1} + \mathcal{B} \mathcal{D}$

Optimal Policy in the example economies

Vertical Chain - Implement a positive output gap which stabilises the upstream sector. This is optimal because distortions are more costly in the upstream sector and moreover it is easier to stabilize.

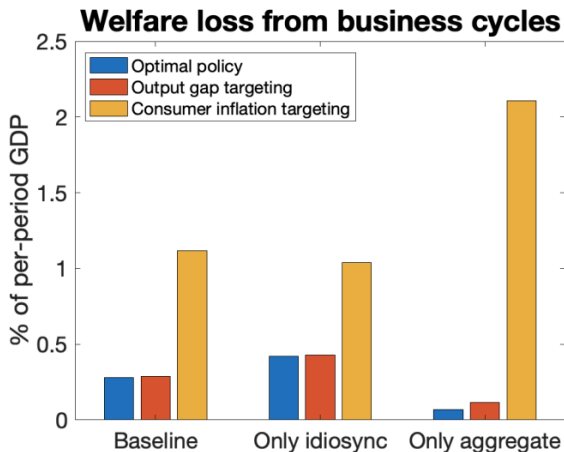


Figure 4: Actual input-output network; different calibrations keep the variance of aggregate output constant

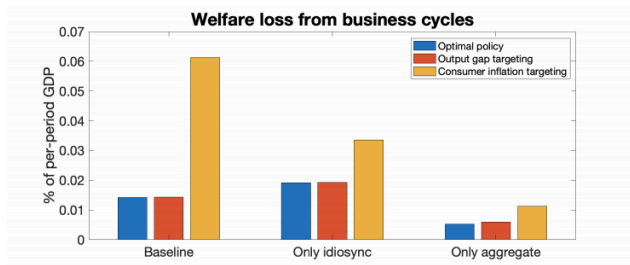


Figure 5: Model with no input-output linkages; different calibrations keep the variance of aggregate output constant

Output Targeting

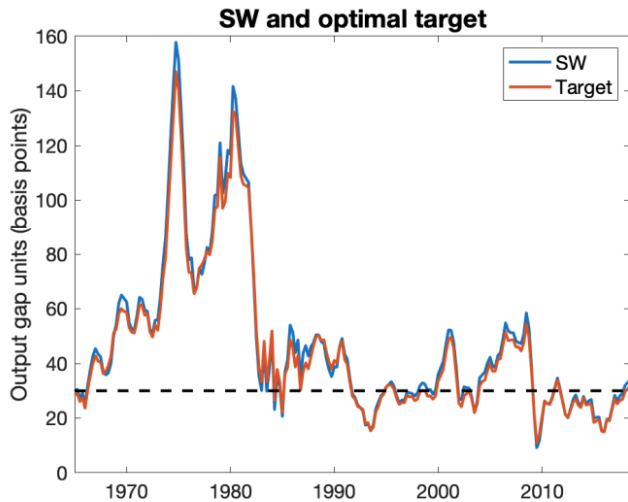


Figure 6: Time series of aggregate inflation (SW) and the optimal policy target

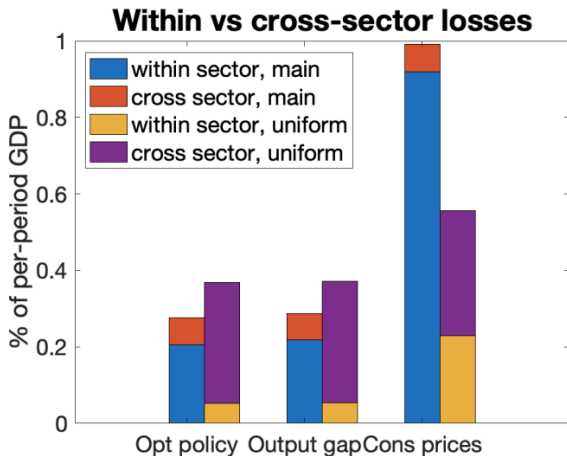


Figure 7: Main calibration: $\epsilon = 8$, $\sigma = 0.9$, $\theta_L = 0.5$, $\theta = 0.001$; uniform elasticities: $\epsilon = \sigma = \theta_L = \theta = 2$

- **Consumer-price Phillips curve is misspecified:** the slope changes with the input-output structure and productivity shocks create an endogenous residual
- **Correct Specification:** aggregates sectoral inflation rates according to sales shares and appropriately discounts flexible sectors. This index has significantly higher R-squared.
- **Monetary policy** cannot replicate the first best, but the optimal policy can still be implemented by targeting an appropriately defined inflation indicator.